RANDOMNESS AND APPROXIMATION

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A Story

- Given a polynomial f(x)
  - Find its roots
- Polynomial time algorithm in degree f(x)

- But…

A Question

- Where does f(x) come from?
- The eigenvalues of real symmetric matrices

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{bmatrix} 3-\lambda & 1 & -1 \\ 1 & 3-\lambda & -1 \\ -1 & -1 & 5-\lambda \end{bmatrix}$$

- All roots real
- Better methods to solve

The Point

- Moore's Law Stops?

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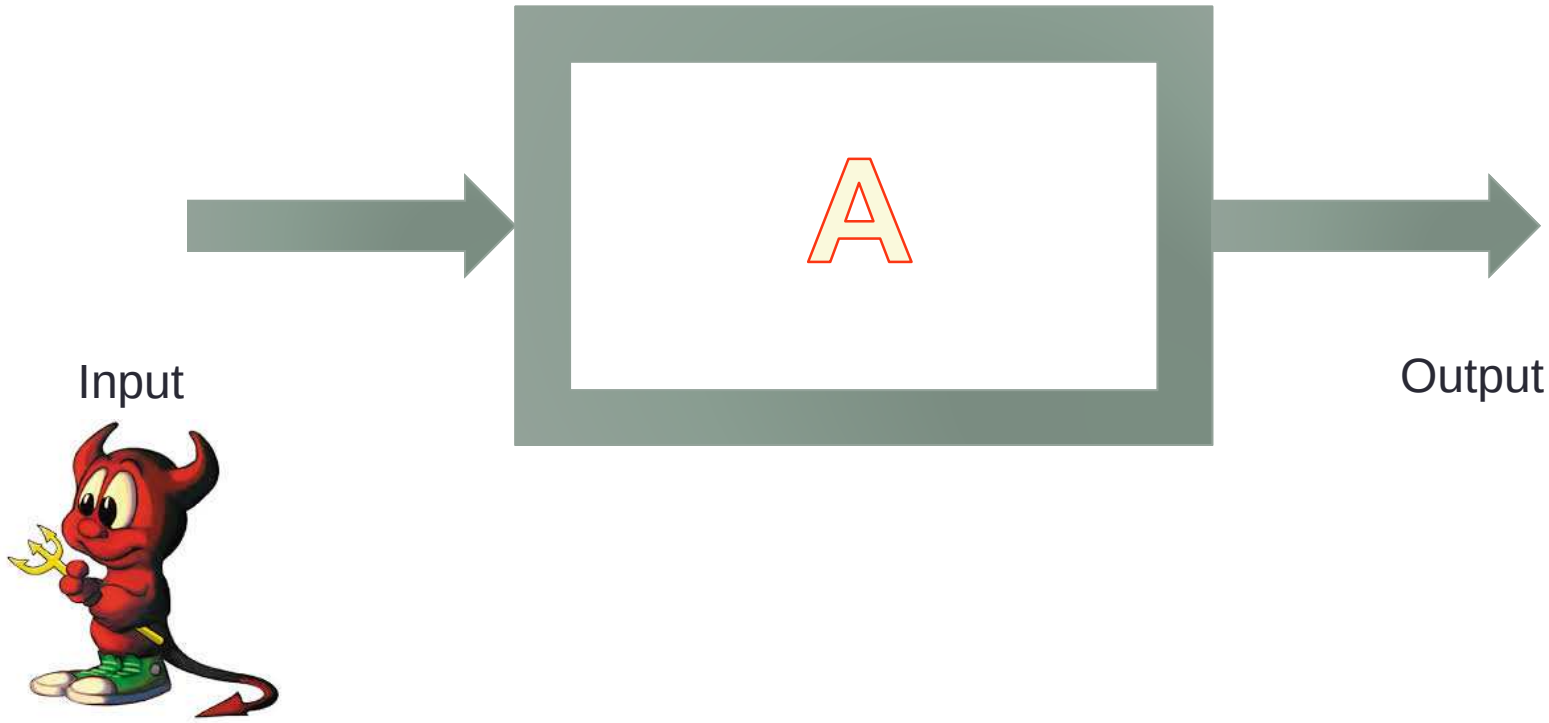
IEEE panel agree Moore's Law via Lithography scaling will be dead by 2035 and explore Beyond CMOS

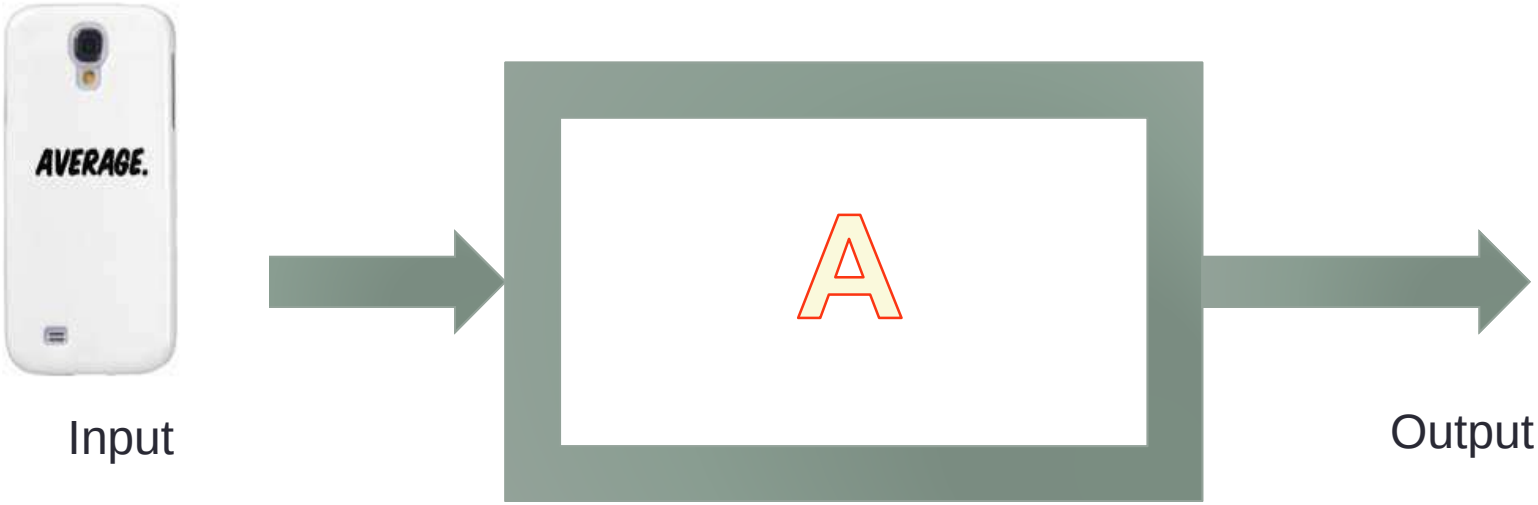
- Must get to core problems

- Perhaps use
  - Randomness
  - Approximation

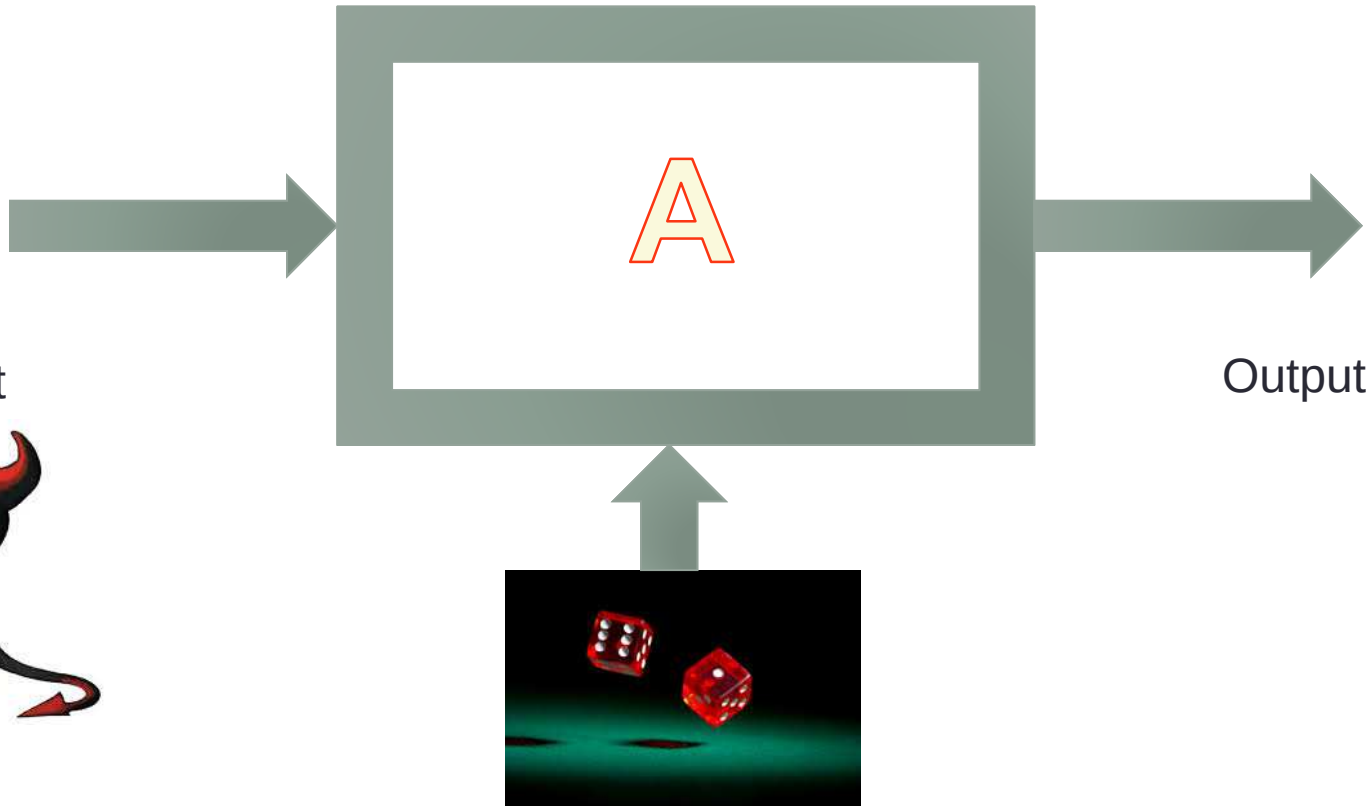
Worst Case



Average



Random



Models

- Worst Case
  - For *all inputs* runs in time T
- Average Case
  - For *most inputs* runs in time T
- Random Case
  - For *all inputs* runs in time T with probability ≈1

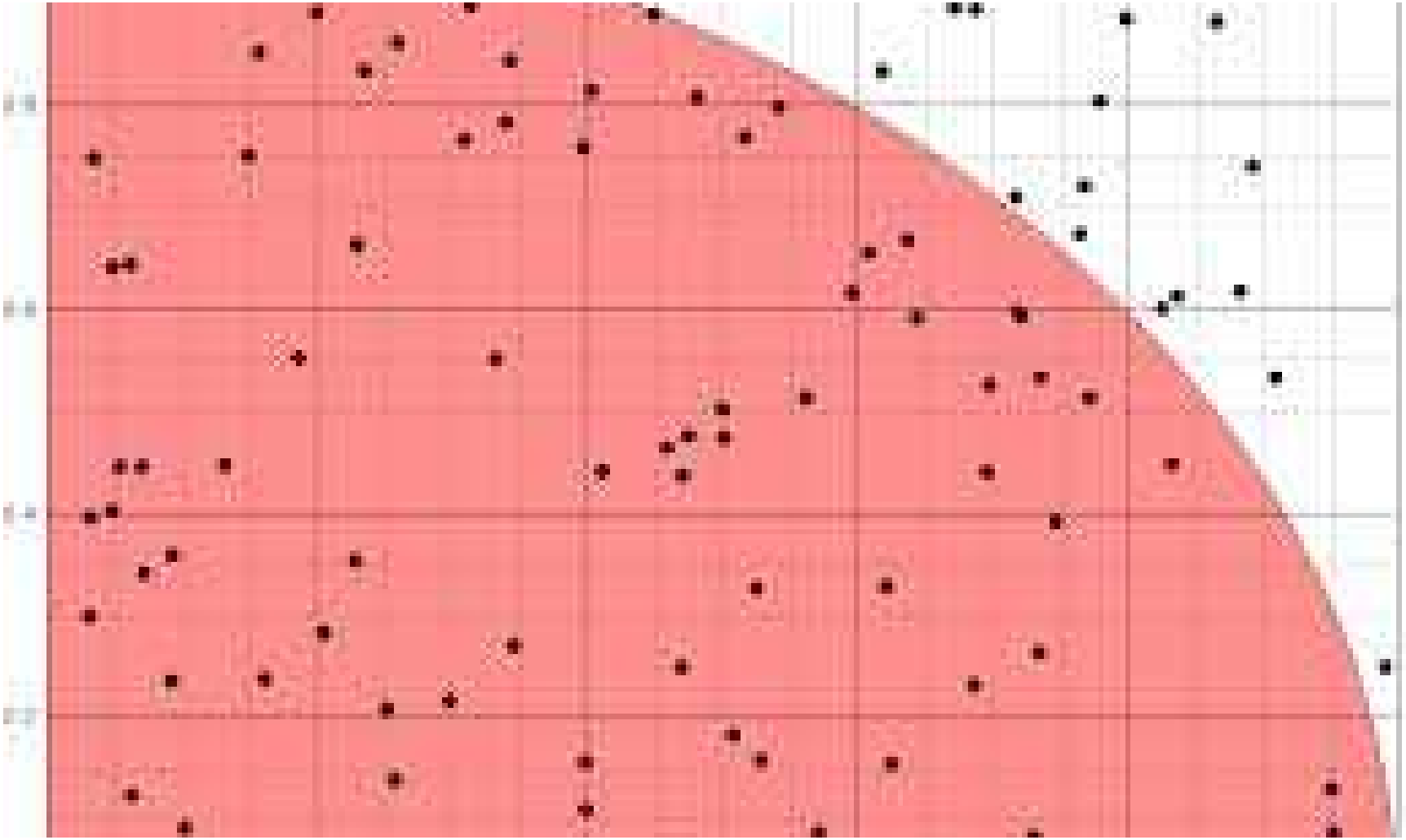
Why Useful?

- Sampling
- Test properties
- Look at small part of information
- Others

Sampling

- Economy
- Weather
- Molecular
- Genetics
- Load balancing
  - Distributed sorting

Sampling



Testing

- Equality
  - $12^{2000001}+7^{46466} = 143^{10100101} + 198^{45546}$?
- Expense
- Can do via randomness

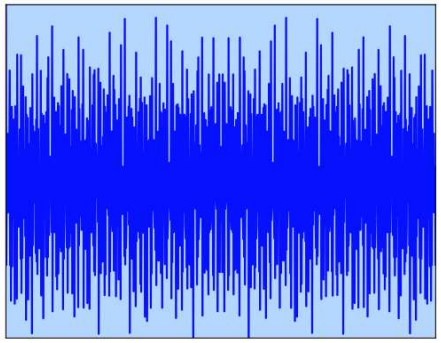
Approximation

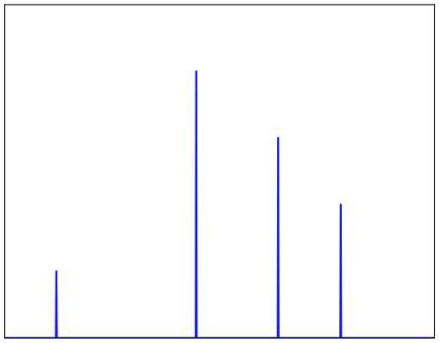
- Compute f(x) or compute g(x) so
  - Distance(f(x),g(x)) is small
- Makes sense
  - Many problems
  - Not all

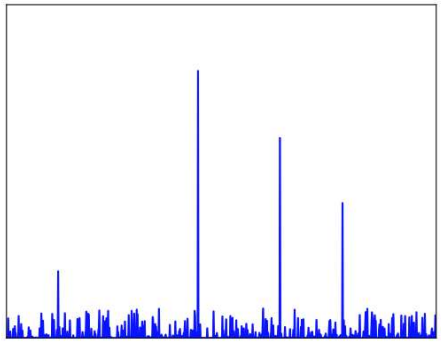
DFT

- Given N points can compute in order N*log*(N) operations
  - Any inputs
  - No randomness

Sparsity







Results

- On N points if k sparse
  - Order operations k*log*(N) random algorithm

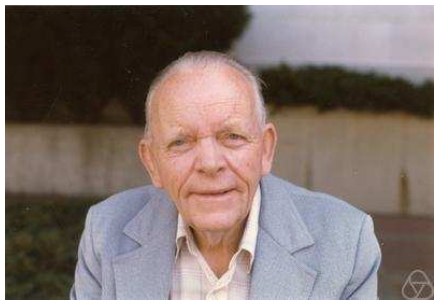
- Approximate sparse

$$\|\text{result} - \widehat{x}\|_2 \leqslant (1+\epsilon) \min_{k\text{-sparse } \widehat{x}_{(k)}} \|\widehat{x}_{(k)} - \widehat{x}\|_2$$

Random Bit Generation

- Issues with generation of random bits
- “Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.”

- “Hard to generate…”



Random Bits

- Algorithms vary
- Some use few
  - Crypto
- Some use more
  - Hashing
- Some use lots
  - Sampling
  - Testing

Issues

- How create culture support random/approximate?
- Harder to debug and test?
- Change view of world?